Getting into an orbit around the voxel planet

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Let h be the height of the player above the planet center and r_p be the planet radius. The gravity at sea level ($h = r_p$) is g_0.

Naive Transformation Approach

Here we assume that every block has the same height. This assumes the game doesn't make use of the aspect-ratio-preserving property of the complex exponential function, so that voxels higher up will look flat and voxels near the core will look stretched.

Forces

Gravity

For the gravity force outside the planet we have Newton's law of gravity:

$$F_G = G\,\frac{m_{player}\,m_{planet}}{h^2}$$

Therefore, the acceleration of the player is:

$$a_G = G\,\frac{m_{planet}}{h^2}$$

Since we know that $a_G(h = r_p) = g_0$, we can conclude that:

$$G\,m_{planet} = g_0\,r_p^2$$

Therefore:

$$a_G = g_0\,\left(\frac{r_p}{h}\right)^2$$

Centrifugal force

The centrifugal force is

$$F_Z = m_{player}\,\frac{v^2}{h}$$

Therefore:

$$a_Z = \frac{v^2}{h}$$

Orbit

For a stable orbit, we want both accelerations to be equal, since their vectors point in opposite directions: $a_G = a_Z$.

$$g_0\,\left(\frac{r_p}{h}\right)^2 = \frac{v^2}{h}$$

Solving for h we get:

$$h = \left(\frac{r_p}{v}\right)^2\,g_0$$

However, the voxel game doesn't think in terms of height above the planet center, but in y above the sea level. With $y = h - r_p$ we get:

$$\boxed{y_{Orbit} = \left(\frac{r_p}{v}\right)^2\,g_0 - r_p}$$

With an Aspect-Ratio-Preserving Transformation

Comparison

Compared to the naive approach, we can't just translate y-Height on the flat map to complex magnitude ($=$ height on the planet) using $h = y + r_p$. Instead, we have to apply the transformation:

$$h = r_p \, \exp\left(\frac{y}{r_p}\right)$$

Both formulas, a_Z and a_G will have to use the new height in order to account for the transformation.

New orbit

Up to the point where we solve for h the calculations above will be the same. With $h = r_p \, \exp\left(\frac{y}{r_p}\right)$ we get:

$$r_p \, \exp\left(\frac{y}{r_p}\right) = \left(\frac{r_p}{v}\right)^2 \, g_0$$

$$\exp\left(\frac{y}{r_p}\right) = \frac{r_p}{v^2} \, g_0$$

$$\frac{y}{r_p} = \ln\left(\frac{r_p}{v^2} \, g_0\right)$$

$$\boxed{y_{Orbit} = r_p \, \ln\left(\frac{r_p}{v^2} \, g_0\right)}$$

Example values

Naive Transformation Approach

Radius r_p $[blocks]$	Velocity v $\left[\frac{blocks}{s}\right]$	Gravity g_0 $\left[\frac{blocks}{s^2}\right]$	Orbit y $[blocks]$
16	4	10	144.00
32	4	10	608.00
64	4	10	2496.00
256	4	10	40704.00
32	4	9.81	595.84
32	4	30	1888.00
32	1	10	30688.00

Aspect-Ratio-Preserving Transformation

Radius r_p $[blocks]$	Velocity v $\left[\frac{blocks}{s}\right]$	Gravity g_0 $\left[\frac{blocks}{s^2}\right]$	Orbit y $[blocks]$
16	4	10	36.84
32	4	10	95.86
64	4	10	236.09
256	4	10	1299.24
32	4	9.81	95.25
32	4	30	131.02
32	1	10	184.59